

Name: _____

Date: _____

M12P HW Section 6.3 Double Angle Identities

$$\sin(a-b) = \sin a \cos b - \sin b \cos a \quad \cos(a+b) = \cos a \cos b - \sin b \sin a \quad \sin(a+b) = \sin a \cos b + \sin b \cos a$$

$$\cos(a-b) = \cos a \cos b + \sin b \sin a \quad \tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b} \quad \tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$

$$\sin 2a = 2 \sin a \cos a \quad \cos 2a = \cos^2 a - \sin^2 a \quad \cos 2a = 1 - 2 \sin^2 a \quad \cos 2a = 2 \cos^2 a - 1$$

1. How do we derive the equation $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$ from $\cos(a+b) = \cos a \cos b - \sin a \sin b$?
2. How do we derive the equation $\sin 2\theta = 2 \sin \theta \cos \theta$ from the identity $\sin(a+b) = \sin a \cos b + \sin b \cos a$?
3. When proving an identity that involves $\cos 2\theta$, how do we tell which of the three equations to use? Should we convert it to $\cos^2 \theta - \sin^2 \theta$, $1 - 2 \sin^2 \theta$, OR $2 \cos^2 \theta - 1$? Are they the same?
4. Prove that $\cos 2\theta = 1 - 2 \sin^2 \theta$ and $\cos 2\theta = 2 \cos^2 \theta - 1$
5. Can we simplify the equation like this? Explain why or why not:

$$\frac{\sin 2\theta}{2} = \frac{\cancel{\sin 2\theta}}{\cancel{2}} = \sin \theta$$

6. Use the double angle identities to simplify each of the following to a single trigonometric function. Show all your work and steps:

a) $2 \sin 0.4 \cos 0.4$	b) $2 \sin 5 \cos 5$
c) $\cos^2 0.7 - \sin^2 0.7$	d) $2 \sin \frac{\pi}{7} \cos \frac{\pi}{7}$
e) $\cos^2 \frac{\pi}{9} - \sin^2 \frac{\pi}{9}$	f) $2 \cos^2 75^\circ - 1$

7. Given that $\sin \theta = \frac{2}{3}$, where θ is in Q1, find the exact value of the following: $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$

8. Given that $\tan \theta = -\frac{5}{6}$, where θ is in Q2, find the exact value of the following: $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$

9. Solve the following equation for $0 \leq \theta \leq 2\pi$ using sum and difference identities:

a) $\sin\left(\theta + \frac{\pi}{2}\right) = \frac{1}{2}$	b) $\cos\left(\frac{3\pi}{2} + \theta\right) + \sin\left(\frac{3\pi}{2} - \theta\right) = 0$
c) $\sin\left(\theta + \frac{\pi}{3}\right) + \sin\left(\theta - \frac{\pi}{3}\right) = 1$	d) $\cos\left(\theta - \frac{\pi}{6}\right) + \sin\left(\theta - \frac{\pi}{3}\right) = \sin \theta$

10. Prove the following identities:

a) $\sin(270^\circ + \theta) = -\cos \theta$	b) $\sin(2\pi + \theta) = \sin \theta$
c) $\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$	d) $\cos(a + m) - \cos(a - m) = -2 \sin a \sin b$
e) $\cos(A + B) \cdot \cos B + \sin(A + B) \cdot \sin B = \cos A$	$\sin(x + y) + \sin(x - y) = 2 \sin x \cos y$

11. Prove the following identities:

a) $\sin(3\pi - \theta) = \sin \theta$	b) $\cos(a + b) + \cos(a - b) = 2 \cos a \cos b$
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c) $\frac{\sin^2 \theta + \cos^2 \theta}{\sin^2 \theta - \cos^2 \theta} = -\sec 2\theta$	d) $\tan a - \tan b = \frac{\sin(a-b)}{(\cos a)(\cos b)}$
e) $1 + \sin 2\theta = (\sin \theta + \cos \theta)^2$	f) $\sin 2\theta = 2 \cot \theta \sin^2 \theta$
g) $\cos 2\theta = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$	h) $\sec^2 \theta = \frac{2}{1 + \cos 2\theta}$

$$\text{i) } \frac{1 - \cos 2\theta}{2} = \sin^2 \theta$$

$$\text{j) } \frac{(\sin \theta + \cos \theta)^2}{\sin 2\theta} = \csc 2\theta + 1$$

$$\text{k) } \frac{\sin 3x}{\sin x} - \frac{\cos 3x}{\cos x} = 2$$

$$\text{l) } 4\cos^2 \theta + \cos(2\theta) = 5 - 6\sin^2 x$$

$$\text{m) } \cos^4 x - \sin^4 x = \cos(2x)$$

$$\text{n) } \cot(2\theta) = \frac{1}{2}(\cot \theta - \tan \theta)$$

O) $\frac{\cot \theta - \tan \theta}{\cot \theta + \tan \theta} = \cos(2\theta)$	P) $\cot^2(2\theta) = \frac{\cot^2 \theta - 1}{2 \cot \theta}$
Q) $\cos^2(2\theta) - \sin^2(2\theta) = \cos(4\theta)$	R) $(4 \sin \theta \cos \theta)(1 - 2 \sin^2 \theta) = \sin(4\theta)$
S) $3 \sin^2 \theta \cos^2 \theta = \frac{3}{8}(1 - \cos(4\theta))$	T) $\sin 2\theta = 2 \tan \theta - 2 \tan \theta \sin^2 \theta$